A Culture of Mathematical Enquiry

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Introduction

This document is a living account of the everyday mathematical experiences of children and teachers as they engage and reflect on mathematics.

Using a pedometer to measure steps at the woods; “I’ve done 1, 2, 8 now”.

Christvie age 4.6

Children at Redcliffe Nursery School are fostered to take risks in their thinking and are given the opportunity to deepen their understanding through intellectual interactions and cognitively challenging resources and experiences. Following children’s interests in this way enables children to invent their own hypotheses and concepts, which are often revisited and refined overtime.

Ribot in Vygotsky (1978) talks of mathematics as an art and a science: the art of articulating and questioning, the science of discovery, uncovering and persisting; therefore we would be suppressing children’s intellectual freedom if we were to shunt their questions and fascinations.

All children are mathematical, it is within the eyes of the observer that enables them to be viewed in this way and can support the extent to which their possibilities open...

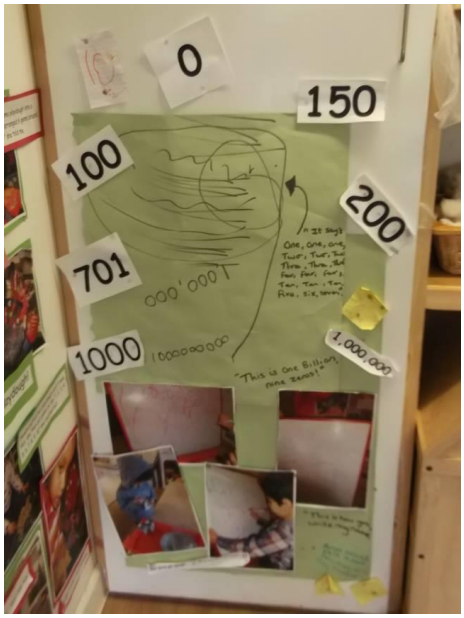
Key group

- Each child’s belonging to a key group means they have a key adult who especially attempts to understand their thinking and interests. This relationship is pivotal to the way the child’s mathematical capacities are viewed and supported.
- It is recognised that children are born into a world which revolves around mathematical principles and children’s experience of these may vary from family to family. This makes mathematics personally significant to children.
- The key group is a collective, made up of individual mathematicians. Some experiences lend themselves to whole group shared collaboration (small group times) and some things may be more pertinent to small subgroups of

the whole group. The key person at Redcliffe strives to create a culture whereby children are engendered to develop their curiosities, ask questions, and request the participation of the key person to share their thinking with. In such a culture, children are active in their use of the key person, at times requesting the involvement of a play partner.

- Our understanding of children's mathematical thinking and desires at Redcliffe has been developed through key persons' skilled observation of each child, also informed by pertinent research, including reading literature. Through professional development we are now beginning to view the broader aspects of mathematics, such as how opportunities in the environment can support understanding and the use of positional language eg. under, over, between etc.

- Creating and changing the nursery environment in response to the *children* rather than through adults' perceptions of what *they* think children should know or for reasons of fulfilling curriculum aims, is supported by research. If children are not exposed to interesting experiences, they will be less likely to discover their own theories, which are essential for higher mathematical thinking, as well being denied their own agency within their learning (Rogers 2010). Responding to and knowing the children allows for links to be made in their mathematics across different modes: language, symbols, and actions. An insightful adult can support the child in further developing these connections.
- As we seek to understand more about children's thinking by being co-partners, discovering with them, we are also enticed into developing our knowledge of the world and understanding how mathematical relationships impinge on almost every daily occurrence. Paley (2007) asserted in her reflections that the key is curiosity, not answers that we model, showing children our desire to also uncover the mysteries of the world. As we seek to learn more about a child, we demonstrate the acts of observing, listening, questioning, and wondering.



- Each key group has a special designated area within the nursery that they can jointly have

ownership; this is an ideal opportunity to seize upon any mathematical threads within the group through displays for and of the children. This may also act as a spark of further influence for the children, for instance photos of different lifts and the floor levels added further discussion for children talking about and recreating a lift out of junk materials. Taking a key group to ride in a lift in the building above the nursery, and the children's reflections after the event, enabled them to explore a wide range of mathematical concepts, including height, the technology of hydraulics and ordinal and cardinal numbers, as well as opening up the thinking and experience for other children too.

- The trusting relationship between children and key person means that mathematical thinking, conversations and experiences can be revisited timelessly, the key person has the whole group in their mind but also the individual children, key groups often share similarities where experiences will appeal to all the children; a ride in a lift and individual children within the group may also be following unique enquiries; for instance a child who revisits and is continuously making connections about tubes, length and attaching materials.

Maths Resources for the key nest area

Each key group has a collection of resources to support mathematical development, facilitated by the key person and influenced directly by the children being able to incorporate mathematical resources themselves as the collection evolves. Maths resources may be personal to key groups and their experiences and the resources which are added support new thinking and interests or can raise new questions. For instance, thermometers were added to a key nest area, after children had conversations about heating, radiators and frost in the winter.

Children's books and number rhymes

There are a broadening range of young children's literature that supports mathematical enquiry. Some books are wonderful at arousing further questions for children, such as 'How high is the Sky'. There are other books with a focus on counting and calculation such 'The Doorbell Rang' and '365 Penguins'. Non-fiction books about all manner of topics are also used at Redcliffe to enrich children's interests, thoughts and conversations between children and adults. Books are also available for children to borrow at home. Books appeal to children's imagination and give a further point of reference in their mathematical thinking. They also build upon children's mathematical literacy and their sophisticated capacity to think, imagine, and converse mathematically.

Number rhymes are engaging for children as they often have repetition, rhyme and are flexible in the way they can be used; children can start from and stop at any numbers and extend them themselves. Number rhymes can be supplemented in different ways too, using fingers, written symbols, the number line, props, or the children themselves. There are a plethora of number rhymes that feature addition, subtraction, multiplication and division. It is important to be mindful of bringing new number rhymes to children and ensuring that a range of mathematical concepts are involved, bearing in mind that addition and subtraction rhymes are most well-known.

Children's questions

- Questions are inherent to mathematics for children, asking questions is a foundation for problem posing and solving. Children's and adult's questions are valued and encouraged at the nursery; questions challenge thinking and make way for the possibility of new thinking and knowledge. When children ask questions, they are inviting new information into their world and want to uncover more. Children's questions can be challenging for adults as they are often multi-layered and complex to answer. This is an exciting part of the practice at the nursery, as the children and adults create knowledge together. For example, children's recurring questions about the flow of the river, which runs parallel to the nursery, led to an adult re-learning about the world from the perspective of the child's curiosities and, in turn, supporting the children to create understanding about tidal rivers, rainfall and water cycles.

- Isaacs (1984) in her research of children's questioning found that children asked and revisited their 'why' question in response to cognitive challenge;

they would ask adults they knew would respond sincerely to their questions and help to satisfy the need of the situation. Tizard and Hughes (1984) found that children persist in their questioning when they seek to deepen their understanding of matters important to them; they termed this 'passages of intellectual search'. We strive to give each child the opportunity to have their passions acknowledged and listened to in this way. This is naturally evolving and is different for each child, but each child has the potential, and we hope we are able to recognise children's searches and be their partner to explore these.

When children observed gas engineers renewing the pipes by their nursery, they asked their key person many genuine questions about how and why this was happening. This later became a feature of children's collaborative play later in the nursery garden; fixing broken pipes, joining pipes together to make a circuit of pipes, with new vocabulary and insight into the underground world. Initial adult involvement and response in children's questions eventually led to children piecing together their knowledge to make their own assertions, creating hypotheses, something which can be viewed as a higher-level skill.

Vygotsky (2004) said how children were freer in their thinking than adults, as they had so many half assembled 'scaffoldings of knowledge' that enabled them to piece together their own theories. An example is when children combine elements of their knowledge to invent a new possibilities e.g. Barley:
"……. a long rubbish grabber that can grab rubbish in space and bring it to earth, crush it and make new things."

The Learning Environment

- At Redcliffe, we want children to have the opportunity to engage with mathematics on many different levels, within a culture of mathematical enquiry. We recognize that this has implications for how the physical space is organised and what resources are made available, on one level and, on another, the opportunities children have to explore mathematical ideas both within their self-initiated play as well as in teacher-guided episodes of collaborative problem-solving. Planning is informed by the children's own enquiries and supplements the basic mathematics provision.

- Displays can both challenge thinking and offer the space for children and adults to engage in a shared understanding of mathematics in genuinely authentic contexts. The subject could come from an aspect of children's-

initiated play or thinking which poses further questions. This may be stimulated by real events, such as building works or experiences gained on visits to factories, working farms and such like. They can take a variety of forms, including temporary installations and messages on white boards as well as wall displays.

- Within a key group space, number lines, at children's height, have numbers that can be moved and placed in any order. These are an important resource and reference for children and adults to refer to when discussing numbers. They are changed as children's understanding of number develops. There is also a low-level clock which acts as a stimulus for discussion about numbers and time, as well as a range of other items to stimulate mathematical thinking, such as calendars, height charts and children's own mathematical graphics.
  Both the inside and outside space is set up to provide plentiful open opportunities for children's mathematics, with an emphasis on children leading their own investigations and play. Adults are there to join with children in their enquiries, offering the potential to extend their thinking through the use of mathematical language and by providing intelligent responses.

- We have open, non-specific resources to allow children freedom in how they use them mathematically. For example, natural materials such as sticks of different lengths, pebbles of varying size and weight and pinecones provide scope for both practical exploration i.e. measuring, weighing, counting, patterns etc. as well as imaginative applications involving abstract mathematical thinking.

- We also have a range of authentic equipment that has inherent mathematical characteristics, such as child level digital hobs, high pressure dial pumps and builders' spirit levels. These types of resources not only offer children

opportunities to think mathematically in authentic contexts but also give them the possibility to reference their experience and knowledge in diverse ways.

- Careful thought is given to making the outdoor environment rich mathematically. The sand area illustrates the pedagogical reasoning behind everyday practice, with over 1 metre depth and extensive surface area enabling developed exploration of a wide range of mathematical concepts eg. when children dig sand to mix with water in different containers to make 'cement' in order to build, using real bricks, they investigate capacity and volume, weight, size, shape, angles and quantity, as well as applying skills in estimation, transformations, addition and subtraction. The timber frame pagoda offers further exploration of height, angles and trajectory, with the possible addition of ladders, ropes, or tubing. Emphasis is placed on equipment that can be used in a variety of ways, is non-static and provides challenge. It is set up by adults, according to their on-going knowledge of the children. For example, extensive use is made of timber and recycled building materials to scaffold children's mathematical dilemmas.

Play

Using different speeds on the blender.

Exploratory play

Play, at Redcliffe Nursery School is viewed as belonging to the child. In this view, the agency, that is, the way of being, seeing and responding in the nursery, of the child is high. Children are free to transfer resources within the environment, hence influencing the provision and culture of working. There is a continual process of development as we learn more about children and their capacity to be agents.

- Play and mathematics have much in common: understanding connections, processes, and possibilities. In play, children are encouraged to follow their own threads of interest, while adults respond to play invitations, bringing new information and resources to the play. The desire to understand the child and be a “real person in the relationship” (Carl Rogers 1983) is a vital part of extending children’s thinking.

- Children may continually revisit a particular aspect of their play which they ‘dissociate’ or separate out from a wider concept, in an attempt to refine understanding, as with exploring “being late” within the broader notion of time. This relates back to Tizard and Hughes’ (1984) ’passages of intellectual search’, where children persist in talking about a thread of enquiry until they are fully satisfied with their understanding.

- Peer influence is acknowledged as intrinsic to children’s mathematical concepts. Among friends, children encounter new ideas and possibilities, create joint meaning making, test out, explain, [‘and enact their perspectives and understanding whilst encountering those of others.

- The open plan environment allows for flexibility in how space is created by adults and children, individually or together. Equally, the choice of exploratory play resources available is influenced by the children’s thinking. Baskets of resources at children’s height allow for children to incorporate the resources fluidly within their play and revisit and reuse naturally. An example would be the growing volume of broken equipment such as phones and

computer keyboards which are available to children for exploration and using in play episodes.

- A mathematically rich environment which is based on exploratory play offers new possibilities - such as syringes, pipettes and calibrated measures added to water play - but it has implications for how we teach. Our teaching requires adults to see the children's mathematical potential in what they are doing and be ready to join their play through genuine interactions, as these episodes happen opportunistically.

A 'tardis time machine': "would you like to go back in time…I'll just put it into the computer…you're going through time". Alfie age 4

Imaginative play

- "Imagination, the ability to think of things possibly being so." (White, in Egan 2007).

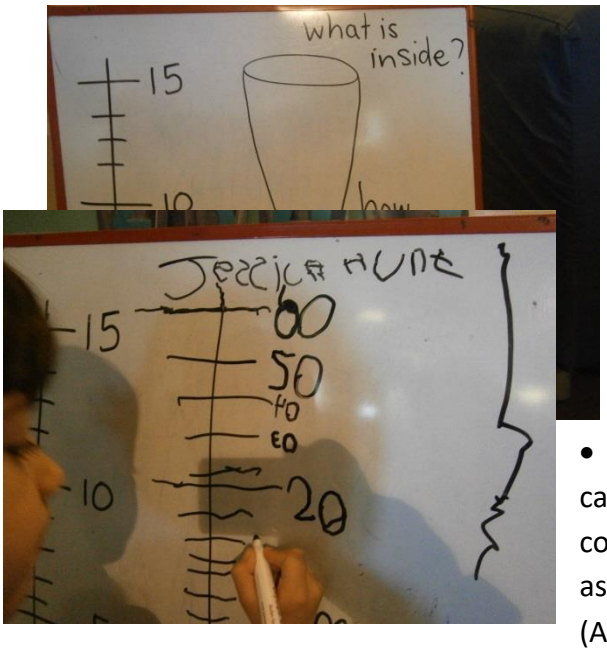
Imagination, "a structure erected in the mind through the agency of new combinations and relationships", is also a fundamental premise of mathematics, particularly in aspects of calculation, the understanding of relationships between numbers and of new combinations. Traditionally, mathematics has been known as a right/wrong study based on recall, and far removed from creativity. However, when the Mathematician George Cantor (1989) stated that "the essence of mathematics is in its freedom", part of his intention was to encourage people to think beyond limits mathematically. Thinking about astronomy, for example, closed questions such as 'how many?' are supplanted by open questioning about infinity.

- In imaginary play, children like to exaggerate aspects of their experiences, "you need to pay me fifteen hundred and sixty pounds" gives children experience of dealing with quantities they have not dealt with in reality. They magnify reality so they can encounter feedback from their exaggeration.

- Children's agency is regarded as pivotal to their meaning making; their views, interests and the capacity they have to follow these is hugely influenced by those around them. This in turn impinges on the views and beliefs they have in themselves. Vygotsky (2004) states that all the elements of children's stories and imaginary play is known to them from their previous experiences, even if the combinations that have applied to create the story are new. These combinations

are also influenced by socio-cultural implications. “The use of the imagination requires: perception, memory, idea generation, emotion, metaphor, intersection, interaction” (Egan 1992:18)

Calculation



Calculation forms a key element of children’s mathematical knowledge. Ways that we foster developing skills in calculation at Redcliffe include:

- Children using their own methods of calculating and solving problems, both in concrete situations involving physical objects as well as through the ‘mental images’ (Atkinson, 1992) they create.
- The teacher’s role in responding to as well as providing problem-solving activities which come from everyday contexts, creating a culture of ‘thinking mathematically’ (Anghileri, 1995)
- Children experiment recording with numbers and abstract symbols, including the use of pictures or letters and words as well as standard mathematical symbols (Gifford, 1990).
- Building on children’s prior knowledge and concepts, noting where children calculate in specific contexts e.g. with a sand timer
- Wide-ranging use of the language of calculation e.g. denoting change of quantities i.e. more, less, how much?, how many?, how often? half, double, a quarter etc.
- Promoting a ‘calculation environment’ by providing a range of resources and displays which offer children the scope and motivation to develop their calculation.

Counting

Counting is a core aspect of calculation. Adults support children's own methods and introduce specific counting contexts or procedures to help develop their knowledge about counting. These include:

- Knowing the number names, either in a number sequence or more in non-numerical contexts, such as on doors or phone numbers.
- Sequencing numbers in order – counting rhymes, with varying start points, counting to keep in time, like in a game of hide-and-seek and developing skills towards counting in 2's / 5's and the 'decade' names ie. 20, 30 etc.
- Organising objects in a way that makes them easier to count.
- Knowing when a number word describes the amount of objects in a set (expressing *cardinality*). This starts with learning of the relevance between a specific number word and a particular reference point e.g. 10 fingers, 5 speckled frogs etc. and develops into an understanding of the way any word within a count has the potential to represent cardinality i.e. by saying at any point how many objects have been counted.
- Counting by pointing at or moving an object; forming a mental picture of an array of objects.
- Estimating or *subitising* the number of objects in a group and making comparisons between different groups.
- Using number words in the context of measuring i.e. where there is a continuum of particular units of measurement
- Saying a different number word for each countable object (counting *one-one*) e.g. the number of stories in a tall building, pieces of apple on a plate etc.
- Knowing that any collection of entities can be counted, whether they be physical or non-physical, homogeneous or miscellaneous
- Understanding that number names are arbitrary, so that the order in which objects are counted is irrelevant.

- Learning about the operations of addition, subtraction, multiplication and division, acknowledging that they are all interrelated processes which can be learnt in any order. Within these operations, specific techniques such as counting on, counting back, counting in groups and sharing out into equal groups apply.

Children's mathematical graphics

This term was coined by a previous head teacher at Redcliffe, Elizabeth Carruthers and her colleague Maulfry Worthington (2003) and continues to grow in profile at Redcliffe, where children's own graphics are highly valued and visible. As the phrase suggests, the written representations belong to the child, representing their thinking whilst engaged in mathematical play or thought. It is understood that children use mathematical graphics to support their own thinking and to communicate their thinking with others. Careful observation and interaction with children helps adults to understand the context and thinking behind the mathematical graphics and how the graphics are supporting the child's thinking. Children's own mathematical graphics helps them to bridge the gap between their own symbols and the abstract language of mathematics. [To find out more about this and about the taxonomy of mathematical graphics, see books by Carruthers and Worthington (2007 and 2010), which can be borrowed from the professional library at Redcliffe.]

"A ghost with 1 2 3 4 5 6 7 smiles, he's got 4 smiles and three cross faces". Tilly, aged 4.2

Assessing Development in Children's Mathematics

We do this through: -

Observations collated in Learning Diaries
Termly reflective evaluations x 6 year
Data collection x 3 year, producing individualised progression wheels
Staff Planning:

- for individual children
- key groups

- whole environment - inside, outside, at the forest.

Dialogue with Parents

Learning Diaries

- The diaries provide a chronological evaluation of each child's mathematical development over the course of the year, in the form of anecdotal and longer observations of children's learning.
- The child's learning is evaluated, and specific development is highlighted.
- The diary is a shared document between the child, family and key person.
- The process of maintaining the diaries encourages the key person to reflect on the mathematical learning of each child, which helps the individual planning for that child.

Termly reflective evaluations

- Outline how children's mathematical understanding has progressed throughout the term
- Experiences that were planned for incorporate the interests of each group as well as of individuals within the groups.
- They reflect the development in the children's mathematical thinking over the course of the year.
- The reflections compliment practitioners' day-to-day responses to the children's own mathematical enquires in terms of how to sustain and develop thinking.

Staff Planning

- Observations of children's mathematics in play are regularly shared and evaluated in end of day discussions, so that possibilities to extend mathematical investigation can be planned for.
- Key mathematical concepts, e.g. in the area of calculation, are highlighted, with references drawn from EYFS Development Matters.

Dialogue with Parents

- Key persons' informal conversations and formal consultations with parents provide opportunities to bridge individual children's home mathematics with nursery learning e.g. sharing stories from home and nursery, both verbally as well as in learning diaries, from which children's mathematical experiences can be enhanced.

Professional development

- Each key person has the opportunity to have mentoring time, within which support can be allocated for writing maths plans, developing a culture of mathematical enquiry and discussing individual children in relation to their mathematical ideas or misconceptions. Staff on professional development courses may have academic mentors to support essay writing and interpretation of theoretical underpinnings and the links between theory and practice.

- The professional library at Redcliffe Children's Centre, has a section on mathematics, these are mostly theoretical books of differing academic levels, which staff at the Nursery can borrow. There are a growing range of books, covering different perspectives about young children's mathematical learning and development.

- There have been and will be continue to be numerous professional development opportunities for staff within and outside the nursery to develop their practice and values about children's mathematical enquiry, from staff meetings, professional development days and courses held by the Maths SLE's.

- The Head teacher is on the Boolean Maths Hub Board and regularly delivers training on behalf of NCETM. Much of this is shared with the staff at the Nursery.

- There are a collection of children's books related to mathematics, particularly numbers and calculation that can be borrowed by settings to use with children to stimulate mathematical enquiries. We have collated a list of the 'top 15 children's mathematical books'.

- Elizabeth Carruthers has been engaged in research and writing about children's mathematics for a large number of years and has recently achieved her doctorate based on early mathematics. She is a source of knowledge, experience and contact for the staff questioning practice and beliefs and has co-written two books on children's mathematics, as well as a range of academic papers. Elizabeth, in partnership with Maulfry Worthington, has created the online 'Children's Mathematics Network' This is a valuable source for people working with children and provides both a way to network with others and use as a reference for latest research and literature on early mathematics education

- In the future, Redcliffe Nursery hopes to extend its work more closely with local primary schools regarding children's mathematics and support settings in opening their views of children's mathematics.

- When Redcliffe Nursery School was a Teaching School it opened the way for Redcliffe to influence practice locally, nationally and internationally.